Déviations pour les temps locaux d'auto-intersections

Rencontre organisée par :
Amine Asselah

06-10 décembre 2010

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Semidirected random polymers: Strong disorder and localization

Vol. 2, n^{o} 1 (2010), p. 47-48.

<http://acirm.cedram.org/item?id=ACIRM_2010__2_1_47_0>

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Actes des rencontres du C.I.R.M.
Vol. **2** n° 1 (2010) 47-48

Semidirected random polymers: Strong disorder and localization

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Abstract

Semi-directed, random polymers can be modeled by a simple random walk on Z^d in a random potential -$(\lambda+\beta\omega(x))_{x\in Z^d}$, where $\lambda>0$, $\beta>0$ and $(\,\omega(x)\,)_{x\in Z^d}$ is a collection of i.i.d., nonnegative random variables. We identify situations where the annealed and quenched costs, that the polymer pays to perform long crossings are different. In these situations we show that the polymer exhibits localization.

We consider a simple random walk $(X_n)_{n\geq 1}$ on $\mathbb{Z}^d$, $d\geq 1$, which moves among an i.i.d. random potential $(\,-(\lambda+\gamma\omega(x)))_{x\in\mathbb{Z}^d}$. We denote by P_x the distribution of the random walk, when it starts from $x\in\mathbb{Z}^d$ (we do not include the subscript when x coincides with the origin) and by $\mathbb{P}$ the distribution of the collection of the i.i.d. variables $(\,\omega(x))_{x\in\mathbb{Z}^d}$. We assume that ω is nonnegative, does not concentrate on a single point and that $\mathbb{E}[\omega^2]<\infty$. Moreover, the parameters λ,γ are considered to be positive. We are interested in the interaction between the random walk and the random potential and how this affects the behavior of the simple random walk. The interaction is described by the Gibbs measure on paths

$$dP^{\lambda,\gamma}_{L,\omega}:=\frac{1}{Z^{\lambda,\gamma}_{L,\omega}}e^{-\sum_{n=1}^{T_L}(\lambda+\gamma\omega(X_n))}dP,$$

where $T_L:=\inf\{n\colon (X_n-X_0)\cdot\hat{e}_1\geq L\}$ and $Z^{\lambda,\gamma}_{L,\omega}:=E[e^{-\sum_{n=1}^{T^{\hat{l}}_L}(\lambda+\gamma\omega(X_n))}]$ is the partition function. Notice that the one end of the polymer is fixed at zero, while the other is constrained to lie on a hyperplane at distance L from the origin. Moreover, the presence of the positive parameter λ induces a drift on the path towards that hyperplane. This presence of a preferred direction explains the terminology *semidirected*. Our goal is to get a qualitative description of the distribution of the end point of the semidirected random polymer, i.e. of the measure $P^{\lambda,\gamma}_{L,\omega}(X(T_L)=x)$, for $x\in\mathcal{H}_L:=\{x\colon x\cdot\hat{e}_1=L\}$. The macroscopic behavior of this measure, as L tends to infinity can be either a *delocalized* (or diffusive) or a *localized* one depending on the strength of the disorder γ (the parameter λ is fixed) or the dimension. The diffusive behavior is established in [3], when the dimension is $d\geq 4$ and the strength of the disorder is low, i.e. γ is small. Here we are interested into the localized behavior. Before proceeding into the detailed statements let us mention that the path properties of the semidirected random polymer are in close connection with what is called the *Lyapounov norms.* These norms measure the cost to perform the crossing from the origin to the hyperplane at distance L. The quenched Lyapounov norm in direction $\hat{e}_1$ is defined as

$$\alpha^*_{\lambda,\gamma}:=-\frac{1}{L}\log Z^{\lambda,\gamma}_{L,\omega}.$$

The path behavior of the polymer appear to be intimately connected to the relation of the quenched Lyapounov norm to the annealed one, which is defined as

$$\beta^*_{\lambda,\gamma}:=-\frac{1}{L}\log\mathbb{E}Z^{\lambda,\gamma}_{L,\omega}.$$

In [5] we show that

Text presented during the meeting “Excess Self-Intersections & Related Topics” organized by Amine Asselah. 06-10 décembre 2010, C.I.R.M. (Luminy).

Theorem 1. *If $\alpha^*_{\lambda,\gamma} > \beta^*_{\lambda,\gamma}$ then $\mathbb{P}-a.s.$ we have that*

$$\limsup_{L\to\infty} \sup_{x:\, x\cdot\hat{e}_1 = L} P^{\lambda,\gamma}_{L,\omega}(x) > 0.$$

This result extends the picture valid in directed polymers [4] and should be contrasted with the one in [3], where the diffusive behavior was established in the regime of parameters where the annealed and the quenched Lyapounov norms coincide. In [5] we also identify situations where the inequality of the norms holds. In particular we have

Theorem 2. *Assume that the disorder ω is nonnegative, does not concentrate on a single point and $\mathbb{E}[\omega^2] < \infty$.*

A. For any $\lambda > 0$, $\beta > 0$ and $d = 2, 3$ we have that $\alpha^*_{\lambda,\gamma} > \beta^*_{\lambda,\gamma}$.

B. The strict inequality between the annealed and quenched norms is also valid in any dimension, if γ is large enough and the disorder satisfies the additional assumptions that essinf$(\omega) = 0$ and $\mathbb{P}(\omega = 0) < p_d$, where p_d is the critical probability for site percolation in $\mathbb{Z}^d$.

Part B. of Theorem 2 is based on a first percolation argument: Assume that β is infinite. If the zeros of the disorder do not percolate, then the origin will be inside a *trap* and any path that has to cross a long distance is doomed to be killed by the potential, while when an annealing is performed the disorder moves around so that to create a corridor, that will allow the path to move at long distance.

Part A. of Theorem 2 makes use of the fractional moment method that has been developed in the context of the random pinning model [1] and has been applied to directed polymer [2].

The heuristics of Theorem 1 are as follows. Consider $\hat{P}^{\lambda,\gamma}_{L,\omega}$ the measure $P^{\lambda,\gamma}_{L,\omega}$ conditioned on the path to stay in between the hyperplanes $\mathcal{H}_0$ and $\mathcal{H}_L$ and $B^{\lambda,\gamma}_{\omega}$ the partition function constrained on this set. The constrained partition function will have the same Lyapounov norms as the unconstrained one. If these norms are different then for N large enough we have

$$\begin{aligned}
\beta^*_{\lambda,\gamma} < \alpha^*_{\lambda,\gamma} \quad &\simeq \quad -\frac{1}{NL}\log B^{\lambda,\gamma}_{\omega}(NL) = -\frac{1}{N}\sum_{n=1}^{N}\frac{1}{L}\log\frac{B^{\lambda,\gamma}_{\omega}(nL)}{B^{\lambda,\gamma}_{\omega}((n-1)L)} \\
&\leq \quad -\frac{1}{N}\sum_{n=1}^{N}\frac{1}{L}\log\sum_{x\in\mathcal{H}_{(n-1)L}}\frac{B^{\lambda,\gamma}_{\omega}((n-1)L;x)}{B^{\lambda,\gamma}_{\omega}((n-1)L)}B^{\lambda,\gamma}_{\theta_x\omega}(L) \\
&= \quad -\frac{1}{N}\sum_{n=1}^{N}\frac{1}{L}\log\sum_{x\in\mathcal{H}_{(n-1)L}}\hat{P}^{\lambda,\gamma}_{(n-1)L,\omega}(x)B^{\lambda,\gamma}_{\theta_x\omega}(L). \qquad (0.1)
\end{aligned}$$

If the measure $\hat{P}^{\lambda,\gamma}_{(n-1)L,\omega}(x)$ does not develop atoms then an ergodic type argument will imply that the sum inside the logarithm is approximately equal to $\mathbb{E}B^{\lambda,\gamma}_{\omega}(L)$, when n is large and therefore the last line in the above will be approximately equal to $\beta^*_{\lambda,\gamma}$, when L is large, leading to a contradiction.

References

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